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Calculation of the reflection coefficient for Rayleigh waves at the edge of an obtuse-angle wedge

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It is a well-known fact that the problem of the scattering of a Rayleigh surface wave by the edge of a wedge does not have an exact solution. This places considerable importance on various approximate approaches to the analysis of the problem. If the wedge angle $\theta$ is close to $180^\circ$, i.e., if the wedge is obtuse-angled (Fig. 1a), the scattered field can be determined according to perturbation theory, where the angle $\theta = \pi - \delta$ complementing the wedge to a plane is treated as a small parameter. However, the calculations are rather formidable even in this limiting case, because each face of the wedge must be assigned its own Cartesian coordinate system.

In the present note we propose a new version of the perturbation theory, aptly called the "round-off method," for calculating the scattering in an obtuse-angle wedge. It essentially entails joining the faces of the wedge by the surface of a circular cylinder (Fig. 1a) whose radius $\rho$ satisfies the condition $k_0 \rho > 1$, where $k_0$ is the Rayleigh wave number. Under this condition, the propagation of a wave in an arbitrary direction over the surface of the cylinder can be regarded to within terms $-1/k_0 \rho$ as propagation along a plane surface. Jumping ahead slightly, we note that the specific value of $k_0 \rho$ is not significant, because $\rho$ is a parameter of our auxiliary construction and, of course, does not enter into the final result. In light of these considerations, the basic problem (Fig. 1a) can be reduced, to within terms $-1/k_0 \rho$, to an equivalent problem in which the domain near the edge of the wedge is regarded as a perturbation $z = f(x)$ in relation to a plane surface (Fig. 1b). This enables us to use the results of numerous studies of scattering by solitary inhomogeneities of a plane surface (see, e.g., Refs. 7 and 8), including the case of oblique incidence, in order to calculate the scattering of a Rayleigh wave by the edge of a wedge. On the whole, our approach is far simpler than direct approximate calculations using two Cartesian coordinate systems.

We illustrate the possibilities of the method in the example of the coefficient of reflection of a Rayleigh wave from the edge of an obtuse-angle wedge in oblique incidence. It should be noted that the analysis of this case is of major interest in its own right, since the case of oblique incidence on an obtuse wedge has never been treated in the literature.

We proceed from the equation for the reflection coefficient $R$ at geometrical inhomogeneities in the form of corrugations or grooves on a plane surface. We write it in the form

$$ R = \frac{k_0(l_0^2 - k_0^2)\nu^2}{k_0^2 f'(k_0)} \left(1 - \frac{k_0^2}{k_1^2} \sin^2 \alpha \right) G, $$

where $\alpha$ is the angle of incidence of the Rayleigh wave on the edge, $f(k_0) = (2k_0^2 - k_0^2)^2 - 4k_0^2(k_0^2 - k_1^2)^2/2(k_1^2 - k_2^2)^2$ is the Rayleigh determinant, $k_0$ and $k_1$ are the wave numbers of longitudinal and shear bulk waves, and $G = \int f'(x) \exp(2ik_0 \alpha \cos \alpha) dx$ is the form factor of the inhomogeneity [the time factor $\exp(-i\omega t)$ is used]. The specific aspect of the investigated equivalent problem for a wedge is contained in the form of the function $f(x)$, which is clearly governed by the relations $f(x) = \pi \sin^{-1}((\theta/2 + |x|/\rho) - 1)$ for $|x|/\rho < (\pi - \theta)/2$ and $f(x) = 0$ for $|x|/\rho > (\pi - \theta)/2$. The derivative of this function $f'(x) = \pi \sin^{-1}((\theta/2 + |x|/\rho) - cot \theta/2 \cot \theta/2 \cot (\theta/2) = \beta + O(\beta^3)$. We substitute the expression for $f'(x)$ in the integral $G$, make the change of variable $y = x/\rho$, and integrate by parts on the intervals $(-\infty, 0)$ and $(0, \infty)$, on which $f'(x)$ is continuous. As a result, we obtain

$$ G = C(\theta/2) k_0 \beta \cos \alpha + O(1/(k_0 \beta^2 \cos^2 \alpha)). $$

Consequently, the quantity $G$ in the first approximation is independent not only of the radius of curvature, but also of the form of the smoothing sur-

FIG. 1. Reduction of the basic problem of the Rayleigh-wave scattering by the edge of an obtuse-angle wedge (a) to an approximately equivalent problem for a perturbed plane surface (b).

FIG. 2. Calculated curves of $|R|/\beta$ vs angle of incidence $\alpha$ of the Rayleigh wave: 1) $\sigma = 0.25$; 2) $\sigma = 0.31$; 3) $\sigma = 0.35$. Experimental values of $|R|/\beta$ for aluminum wedges ($\sigma = 0.35$) with various vertex angles: 4) $\beta = 20^\circ$; 5) $\beta = 60^\circ$. 

face, and it is determined entirely by the jump of the derivative \( |\Delta| \) or, equivalently, by the angle \( \beta \). Using the expression obtained above for \( |\Delta| \) and confining the problem to the first approximation with respect to \( \beta \) and \( 1/kp \cos \alpha \), we obtain the required expression for the reflection coefficient from Eqs. (1) and (2) in the case of a Rayleigh wave at the edge of an obtuse-angle wedge:

\[
R = \frac{\sin^2(Lk/2kR) - \frac{1}{2k} \cos \alpha}{\cos \alpha (1 - \frac{1}{2} \sin \alpha)} \tag{3}
\]

It follows from Eq. (3) that the coefficient \( R \) depends on the frequency and is determined only by the angle \( \beta \) (in radians), the Poisson ratio \( \sigma \), and the angle of incidence \( \alpha \). For \( \alpha = 0 \), i.e., in the case of normal incidence, Eq. (3) coincides with the results obtained in Ref. 4, attesting to the validity of the proposed approach. Curves of \( |R|/\beta \) as a function of \( \alpha \), calculated for three values of \( \sigma \), are shown in Fig. 2. As in the case of reflection form inhomogeneities in the form of corrugations and grooves on a plane surface, the obtuse wedge has an angle of incidence \( \alpha = \sin^{-1}(k R/k_0) \) for which the quantity \( |R|/\beta \) is equal to zero (forming an analogy with the Brewster angle). After passing through zero, the function \( |R|/\beta \) grows quite rapidly and tends formally to infinity as \( \alpha \to 90^\circ \). This result mirrors the fact that, Eq. (3) is not valid for close to grazing angles of incidence, because \( \cos \alpha \to 0 \) in this case, and it is no longer possible to restrict Eq. (2) to the first term. We recall that the limits of validity of Eq. (3) with respect to the angle \( \beta \) are determined by the condition \( \beta < 1 \). What this means in practice is that the angle \( \beta \) must not be greater than \( 25^\circ - 30^\circ \).

We have measured the angular dependence of the modulus of the reflection coefficient \( |R| \) for two aluminum wedges \( (\sigma = 0.35) \) with angles \( \beta = 20^\circ \) and \( 40^\circ \) in order to confirm the result (3) experimentally. The measurements were carried out in the pulsed regime (with a pulse duration of 5 \( \mu s \)) at a frequency of 2.1 MHz according to the procedure described in Ref. 9. The results of the measurements are shown in Fig. 2 alongside the corresponding theoretical curve (for \( \sigma = 0.35 \)). We see that the experimental points are in good agreement with the calculations in the case of a wedge with angle \( \beta = 20^\circ \) (or \( \beta = 150^\circ \)) and, as expected, they agree with the theory only qualitatively in the case \( \beta = 40^\circ \) (or \( \beta = 140^\circ \)).

5. V. V. Krylov, Akust. Zh. 21, 754 (1975) [Sov. Phys. Acoust. 21, 425 (1975)].

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Field of a spherical focusing transducer with arbitrary aperture angle

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Two methods are used primarily for the calculation of convergent wave fronts. The first method calls for the integration in the Helmholtz–Huygens integral to be carried out over the surface of the wave front. The linear dimensions of this surface are assumed to be large in comparison with the sound wavelength in the medium, so that the surface of integration can be represented by a large set of practically plane sections, whose dimensions are also greater than the wavelength. The resultant field represents the sum of the fields of these plane sections, which radiated independently of one another. Within the framework of this method, O'Neill2 has derived an exact equation for the field on the acoustic axis of a spherical focusing radiator; the equation is valid for a transducer having a small aperture angle and a linear dimension that is large in comparison with the wavelength.

In the second method,3 a plane surface is introduced for the convergent wave front, the true boundary conditions on the surface of the transducer are translated into boundary conditions in the plane, and the field is calculated by integrating over the plane surface. This method has the advantage that an exact integral solution of the Helmholtz equation exists for a plane on which the particle velocity or its potential is specified, whereas such a solution has not been found for an open spherical front. Lucas and Muir4 have used this method to obtain the field distribution for a spherical transducer with a small aperture angle. The results obtained in Refs. 2 and 4 agree in the vicinity of the center of curvature of the transducer.5

It was assumed in the field calculations in Ref. 2 that the wave from each of the plane sections into which the transducer surface is partitioned is not diffracted by the adjacent sections. In Ref. 4, on the other hand, the aperture angle was assumed to be small merely to facilitate the calculations. We therefore use the method of transferring